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A Review of the Severity Index

John Versace
Ford Motor Co.

Abstract

The SAE Severity Index is supposed to be an approximation to tolerance limit data, but there are incongruities in its derivation which renders the formula unsupportable. The same logic on which the Severity Index appears to be based can be used to support a wide range of possible values for the exponent on the acceleration, including infinity. This inconsistency results because necessary distinctions have not been made between: the formula for a fitted approximation to the tolerance limit data, the scaling of severity as such, and the measure of the acceleration magnitude of a pulse, the "effective acceleration." It is recommended that a formula which more literally follows from the tolerance limit data be adopted. In the long run, however, it is believed that a much more appropriate measure of injury severity would result from processing head impact data in such a way as to reflect the probable degree of brain injury.

Predicting Fatalities and Injuries from Tests

Human Tolerance - The degree of human tolerance to head impact is not well established, although there has been as much or more work in this area of human tolerance as in any other. Laboratory studies to find out how hard an impact can be before it produces a serious injury just cannot be done on people the way other biologists might study, say, insects.

When a toxicologist wants to find out how much chemical stress an insect species can tolerate, he does a very direct experiment. He puts 100 insects in each of 7 jars and then he sprays them with the insecticide, incrementing the dosage from jar to jar. He determines the tolerance level by counting how many insects died for the dosage applied, the test results being known as a "bioassay." Two types of findings make up the bioassay: the tolerance limit or threshold dosage for some designated degree of response to occur, and the scaling of the response relative to the dosage–that is, how many more insects are killed by an x% increase in dosage–once the tolerance limit or threshold has been exceeded. The distinction between these two concepts of response sensitivity should be kept in mind, for one of the main points to be made in this paper will bear upon it.

In contrast to such a direct approach, human tolerance to impact can be studied only by indirect methods. For example, subinjurious tests have been run with human volunteers. Such tests at least reveal that the tolerance is some level greater than what was voluntarily endured. Also, human cadavers (usually of quite old men, long embalmed medical school rejects) and live animals have been assaulted in various ways and the resulting damage inspected. But these methods are of such limited value, and the studies so few and of such small scope, that present knowledge of human tolerance to impact is quite incomplete and tentative.

Calibration of Test Devices - Even if human tolerance information issuing from the biological laboratories were accurate and comprehensive, there would still be significant problems of interpretation because practical testing by a vehicle manufacturer requires simple and repeatable test methods and impacters, not cadavers or live animals; after all, his purpose is to test the vehicle components, not the cadavers or even the mechanical impacters. Ideally, for practical testing purposes there should be a "calibration curve" which relates the test readings obtained with specific impacters to the percent of occupants who will survive when exposed to real accidents.

The kind of information needed to construct such a calibration is almost totally lacking. As a result, there is a strong tendency to act as if the numerical values of force and deceleration measured in vehicle tests have the same significance as those observed in the biological laboratories where the response of animals or cadavers was originally studied. There is little reason to believe that the numbers found in a biological experiment and those registered in a mechanical impacter used for testing instrument panels will correspond. In fact, there is a three-way equivocation: Real world impact experience may be reflected only poorly by the findings from the biological laboratory; the biological laboratory's procedures in turn are likely to be only poorly duplicated in the test engineer's devices. As a result, the test engineer's measurements are not likely to relate to real-world accident experience except by a considerable margin of uncertainty. While test-derived indexes of impact severity have been of practical use in facilitating product design decisions, correlation with the eventual count of traffic injuries and fatalities is largely unknown. Because of the lack of information, methods of measurement put forth in this paper are also subject to much of the same criticism.

Head Injury Tolerance Limit

The basic criterion for most evaluations of head impact trauma is the Wayne State University Tolerance Limit. It is shown in Fig. 1, which is reproduced from SAE J885a. Any point on the curve of the Tolerance Limit is supposed to represent the same threshold of injury as any other point. The curve shows that very intense head acceleration is tolerable if it is very brief, but that much less is tolerable if the pulse duration exceeds 10 or 15 ms.

This tolerance limit is probably not accurate. The impact data on which it is based were sparse and they were not very representative. It resulted from animal tests involving frontal hammer blows and air blasts to the exposed brain, and from drop tests of human cadaver heads. No direct tests using pulses longer than some 25 ms were involved in determining the tolerance limit, but consideration was given to assumed levels of deceleration experienced by humans in events such as free falls.

An alternative tolerance limit criterion might be based on the summary compilation of tolerance data, based mainly on military research, published in 1959 by A. M. Eiband (1)*. His summary for frontal deceleration is reproduced in Fig. 2. Note it is plotted in log-log coordinates; it is otherwise analogous to the Wayne State findings in Fig. 1.

Because Fig. 2 covers a much broader range of pulse durations than do the Wayne State data, it has the appearance of presenting a more valid overview of human tolerance. But the data comprising it are probably less accurate, for head injury severity purposes, than that obtained at Wayne State, as will be discussed below. Proposals have been made, notably by Gadd (2) for a criterion to be based on the Wayne State data and the above findings, with emphasis on the latter. Gadd's suggested criterion is represented by a straight line approximating the data and identified as "$1000 = TA^{2.5}$" shown superimposed on Fig. 2.

The approximation shown in Fig. 2 would seem to be compatible with the

*Numbers in parentheses designate References at end of paper.

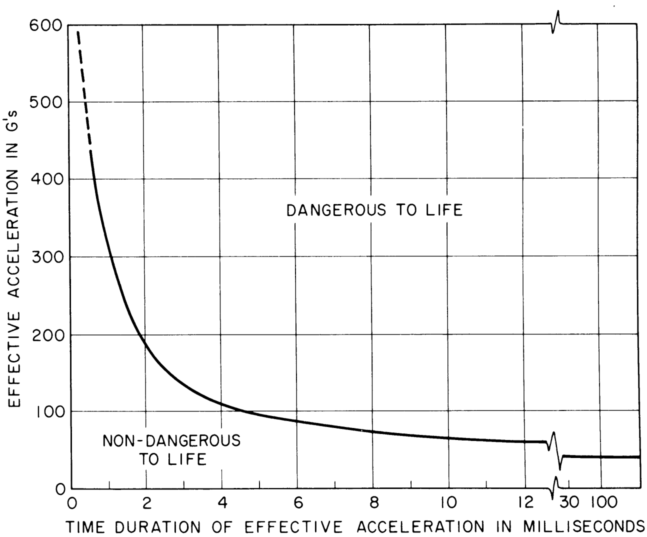

Fig. 1 - Impact Tolerance for the human brain in forehead impacts against plane, unyielding surfaces

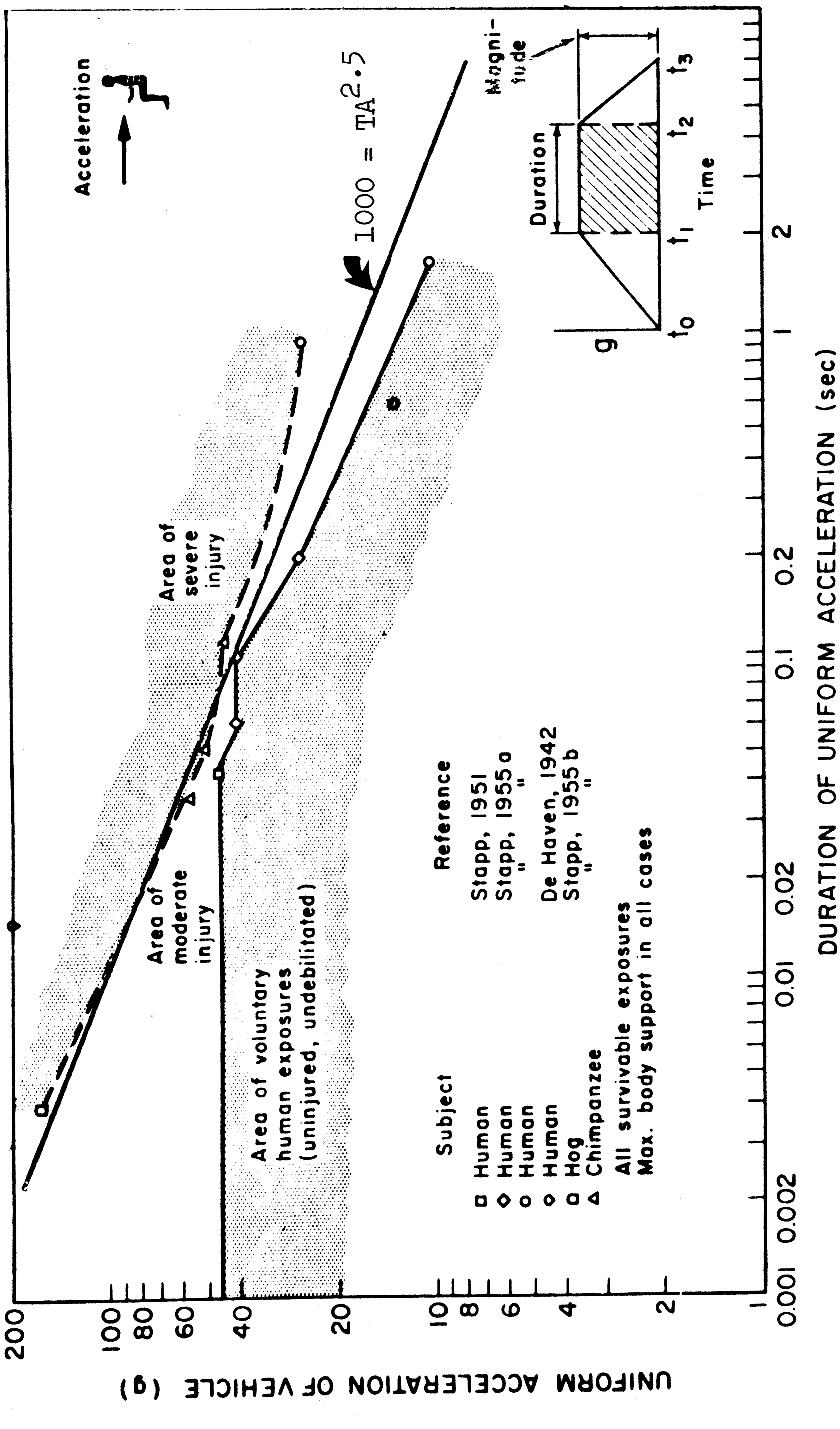

Fig. 2 - Frontal deceleration

Wayne State Tolerance Limit data, but that is not necessarily so because the acceleration measures in the two studies are not commensurate. They are not comparable for at least three reasons:

1. The Wayne State data refer to accelerations measured on the body of the subject experiencing the acceleration. In that respect, these measurements bear a closer relevance to crash testing because the response in the head of a crash dummy or analogous impacter is recorded. By contrast, the military data summarized by Eiband are expressed in terms of the deceleration of the seat on which the test subject rode, not the dynamic response of the test subject. In view of the almost certain amplification of acceleration at the subject's head because of the dynamic action of his body with the restraining devices, the experienced accelerations were undoubtedly greater than as plotted. Consequently, we should expect that the true tolerance is higher and that the fitted approximation should likely have a considerably shallower slope in the 0.1 s region then is shown–if the ordinate of the graph were to represent g experienced by the subject rather than the vehicle, as in the Wayne State data.

2. A second source of difference arises because tolerance data for the very long durations more likely relate to a different mode of injury, fluid displacement, rather than the mechanical type of damage we expect from the shorter duration impacts in car crashes. Therefore, tolerance data recorded for the longer durations should not be given as much weight; to do so would cause the fitted approximation to be skewed away from its appropriate level at the shorter or intermediate durations which are more pertinent to the automotive case. This is a common problem with straight line fits of data over a wide range. Forcing the fit to be best on the average over the whole range can result in a worse fit in a local region of high interest. Wide ranges often span different modes of response.

3. The third source of incomparability to the Wayne State data is the very definition of acceleration level, the value that is plotted on Figs. 1 and 2. The "level" of acceleration must be abstracted from the acceleration pulse. Should it be the peak value? The situation is simple in the case of the military data because all the pulses, being programmed vehicle input rather than occupant response measures, were more or less rectangular waves of constant acceleration level. On the other hand, the accelerations recorded in the Wayne State data tended to be somewhat rounded-off triangular pulses. As a result, the ordinate of the Wayne State graph is stated in units of "effective acceleration." Patrick, et al., state (3):

> "In the general case of impact to a padded surface the acceleration pulse is approximately triangular or sinusoidal, so an effective acceleration value is used. The ordinate of the Tolerance curve of Figure 1 is Effective Acceleration which is based on a modified triangular pulse in which the effective acceleration is somewhat greater than half the peak value. Therefore, triangular or sinusoidal pulses of equal area and higher peak magnitude are

in accord with the experimental evidence from which the Tolerance curve is derived."

In a similar manner, SAE J885 (March 1964 issue) in addition to presenting a weighted-impulse formulation, included the following definition for effective acceleration:

"2.4.2 Impulse Area Criterion - Area under the acceleration-time curve, pressure-time curve, or force-time curve may sometimes provide a useful approximation to its injury potential. This assumes equal importance of the ordinate and its time duration in their contribution to tissue damage.

"Under this criterion, the area under very sharp spikes of the acceleration-time wave is generally negligible and is therefore ignored. The problem is to find the effective acceleration magnitude, which, when multiplied by the

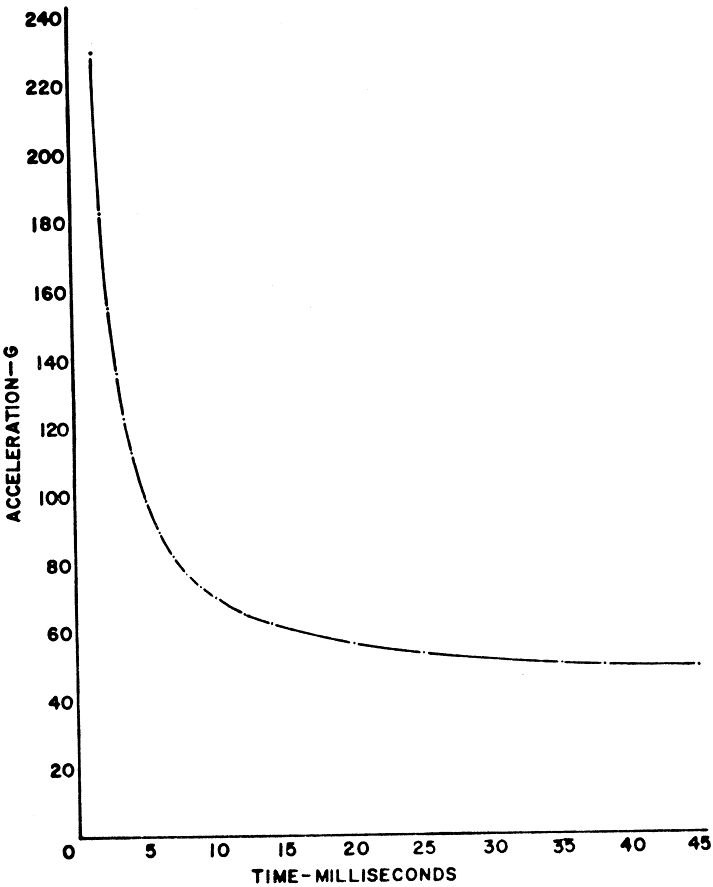

Figure 1 - Acceleration-time tolerance curve for forehead impact to a hard, flat surface

total time duration, will give the area under the curve. Therefore, the following guides should aid in arriving at an effective value for acceleration.

"(a) If the acceleration-time trace has an essentially flat plateau (ignoring any spikes present) the magnitude of the plateau is the effective acceleration.

"(b) For cases in which the acceleration-time trace approximates a half sine or is very irregular in shape, the effective acceleration is the area under the curve divided by its time duration."

Recently, a DOT contract (FH-11-7288) at the University of Michigan (4) revisited the data on which the Wayne State University Tolerance Limit is based. Drawing also upon the results of additional animal studies and on an analytical formulation related to strain in the skull, they concluded that the asymptotic tolerance is greater than is depicted by the Wayne State Limit (Fig. 3). Patrick, et al., had said that too (3): if concentrated loads could be avoided, the effective acceleration asymptote of 42 g was probably unrealistically low—"Consequently, when an impact to padded surface occurs a higher effective acceleration limit is in order. It is estimated that a 60 to 80 g limit is a reasonable value." Presumably, Patrick, et al., refer to the kind of blow which results in an effective acceleration of 60-80 g in real people, not in the heads of crash test dummies.

There are two technicalities associated with the Wayne State Limit which should be noted, as they will be of particular significance in the rest of this paper. The first technical point was covered above, and relates to the meaning of "effective acceleration." The second technical point deals with the distinction

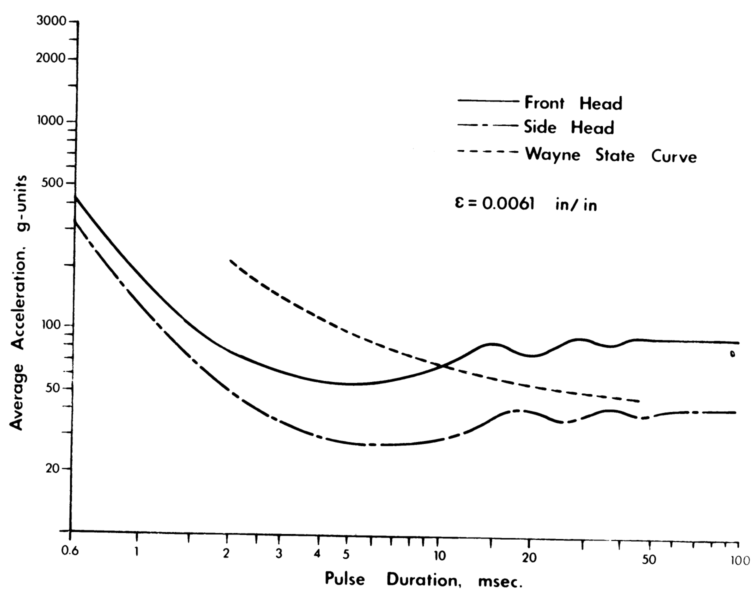

Fig. 3 - Maximum strain criterion for humans

between a tolerance limit or threshold and the scaling of the response once the threshold has been exceeded, a point emphasized in the opening paragraphs of this paper. The Wayne State Limit or any mathematical approximation to it, or any alternative curve brought forth as a replacement for it, represents only the boundary between acceptable and unacceptable levels. It in no way provides a basis for scaling the injury severity or hazard to a victim as the pulse amplitude is varied, for the same pulse duration. A later section of this report will deal with that matter in more detail.

Mathematical Approximation to the Tolerance Limit

The acceleration-time tolerance values, such as seen in the Wayne State Tolerance Limit curve in Fig. 1 and as collected by Eiband, can be plotted on log-log coordinates. In so doing, the data will tend to fall into a somewhat linear configuration. Gadd observed that a straight line on log-log coordinates fits the available data reasonably well.

A straight line in log-log form has the following equation:

$$\log A = m \log T + \log k \tag{1}$$

the slope of the line is m, and (log k) is a constant intercept. The best-fitting line at which Gadd arrived (as shown in Fig. 2) has a slope m of −0.4. The value for k is determined by setting T = 1 (in which case log T = 0); at T = 1, the value of A—as seen on the plotted graph—is 15.85 g. Substituting these into Eq. 1 gives the value of k:

$$\log 15.85 = -0.4\,(0) + \log k$$

$$15.85 = k$$

Putting these constant values of m and k back into Eq. 1,

$$\log A = -0.4 \log T + \log 15.85$$

$$\log A = \log T^{-0.4} + \log 15.85$$

$$\log A = \log\,(15.85\,T^{-0.4})$$

$$A = 15.85\,T^{-0.4}$$

$$TA^{2.5} = 15.85^{2.5} = 1000$$

Thus, the equation for Gadd's line, the *boundary* between acceptable and unacceptable is

$$\boxed{\begin{aligned} 1000 &= TA^{2.5} \\ 15.85 &= AT^{0.4} \end{aligned}} \qquad (2)$$

There can be no cavil with this equation; it is the only equation of that line. The SAE Severity Index, which bears a superficial similarity to this equation, is not the same thing and cannot be derived from these data or from the line which the above equation represents. However, there are two uncertain aspects to the above equation: First, the line it represents is probably not the most appropriate fit and hence the formula would be different if a better fitting line were to be found. And, there is no reason why the fitted line should have to be straight. While all impact tolerance data are rather dubious, this line is skewed by some that are particularly dubious, those for the longer pulse durations. The second uncertain aspect of the equation relates to the definition of the level of "A." Patrick, et al., (3) regard "A" to be some kind of average, the "effective acceleration." It is not certain whether the Wayne State data can even be plotted on the same graph with the military data because of the equivocal identification of the ordinate variable.

The formula can be used in lieu of a tolerance limit graph in establishing whether any observed impact exceeds the tolerance boundary or not. But it can only serve as a "go/no-go" gage. For example, suppose a head deceleration of 46 g and 70 ms were recorded. The point (46, 0.07) could be plotted on the tolerance graph and its position compared to the Tolerance Limit line. Alternatively, either form of the above formula could be used to calculate whether the pulse exceeded the Tolerance Limit:

$$(0.07)(46)^{2.5} = 1005$$

or equivalently,

$$(46)(0.07)^{0.4} = 15.88$$

The first solution exceeds the constant 1000, the second exceeds 15.85; by either version of the formula, the pulse is seen to have a value that places it slightly above the plotted Tolerance Limit line.

Eq. 2 should not be invested with more meaning than it has. It merely specifies the criterion line. That line is supposed to represent a boundary, or threshold, and therefore can in no way represent a scaling of injury severity.

Severity Scaling versus Tolerance Limit

Constants have been taken from the formula of the Tolerance Limit line for severity scaling purposes, a use which cannot be justified conceptually or mathematically. The SAE Severity Index, for example, expropriates the constants "1000" and "2.5" from the boundary formula and inserts them into a different formula:

$$1000 = \int a^{2.5}\, dt \tag{3}$$

It was shown above that there are two different ways of depicting the same line:

$$1000 = TA^{2.5}$$

$$15.85 = AT^{0.4}$$

If the exponent of "A" is somehow indicative of what the severity scaling should be, why not pick the second form of the equation, where the exponent is unity? In fact, the very same Tolerance Limit line can also be depicted by any number of equivalent equations, with the exponent "A" taking on any value desired. For example,

$$10^6 = A^5T^2$$

$$3.98 = \sqrt{A}/T^5$$

are also equivalent, and still refer to the same line.

Now consider another example of this distinction, the distinction between the parameters of a boundary and the scaling of severity, by examining tolerance limit lines that are somewhat different from one another but within the range of normal variation. Assume we have a test result whose acceleration level and duration happen to place it right on the $1000 = TA^{2.5}$ line. We would conclude it just fails being acceptable. Now, suppose that we were to discover some new biological tolerance data, data which cause us to seek a better fitting Tolerance Limit, one whose equation turns out to be, say, $K = TA^{3.6}$, a value which is quite plausible. Let us also suppose the newly fitted line happens to go right through the same test point on the graph, pivoting at that point (not a necessary assumption, but it simplifies the discussion.) Now, in both cases the data point is right on the criterion line, but in the first it involves an acceleration exponent of 2.5 and in the second an exponent of 3.6. The pulse has not changed, so its seriousness is no different just because of a change in shape of the boundary. (To see the effect of exponentiation, note that if $A = 50$, $A^{3.6}$

is about 74 times larger than $A^{2.5}$!) The exponent in the tolerance limit equation has nothing whatsoever to do with the seriousness of the pulse. (See Fig. 4.)

The line corresponding to the equation $1000 = TA^{2.5}$ is not the only plausible line. In fact, it does not fit tightly to the Wayne State Tolerance Limit. The fit is better in some parts of the curve than at others. Table 1 shows constants for several alternative fitted curves.

These are all plausible descriptions of the available tolerance limit data. In fact, in the limited range of 30-100 ms, a near asymptote condition exists in the Wayne State and in the University of Michigan tolerance criteria, as seen in Figs. 1 and 3. Although a flat slope may not be strictly accurate, it is far from being implausible as a simplifying assumption over a narrow range. The exponent gets rapidly larger—approaching infinity—as the curve gets more hori-

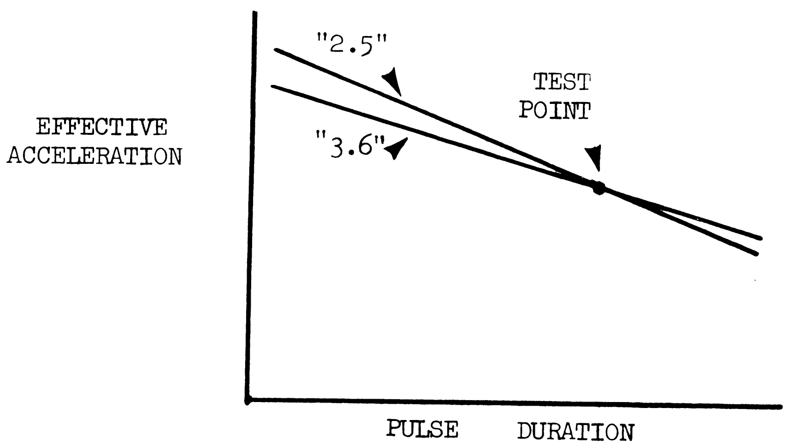

Fig. 4 - Acceleration versus pulse duration

Table 1 - Constants for Several Tolerance Curves

Constant	Power	Comment
1000	2.5	Above the Wayne State Tolerance Curve beyond 2 ms. Goes through a reference point of about 23 g at 400 ms
3780	2.9	Only slightly above Wayne State Tolerance Curve beyond 4 ms. Also goes through same reference point
9580	3.2	Extremely tight fit to Wayne State Tolerance Curve from about 7 ms on. Ditto on reference point
30.8	1.9	Extremely tight fit to Wayne State Tolerance Curve for less than 12 ms. Diverges elsewhere

zontal. The terminal slope of 20 g per decade of time duration implied by extension of the WSU curve corresponds to an exponent of 6. If an exponent is to be expropriated from the formula of the tolerance boundary, and used for severity scaling purposes, which one would be chosen and on what basis would the choice be made? The choice will make quite a difference in practice. But none of the possible exponents are really appropriate because the basis for the scaling of severity must come from some other source than the tolerance limit.

This paper has tried to show that the definition of a Tolerance Limit, or boundary, cannot be used as a definition of a severity scale. In view of the strong intuitive appeal of the SAE Severity Index (5),

$$\int_0^t a^n dt,$$

its apparent simplicity, and the familiarity it has in practice, there may be reluctance to examine critically the logic that produced it. It was stated that the SAE S.I. was not derivable mathematically from the tolerance data. How then, was it produced? The rules which lie behind the SI formula seem to be these:

Rule 1: Fit an approximation to the available tolerance data by drawing a straight line on the graph of log acceleration versus log time duration. (Log A versus log T; the line is drawn so it passes through 100 g at 0.01 s and 300 g at 0.001 s, a plausible fit given that the tolerance data are accepted at face value and that the acceleration levels are established with some consistent method of measurement. This line is shown in Fig. 2. Its equation is $1000 = TA^{2.5}$.)

Rule 2: Find the slope of that line. (Slope is −0.4.)

Rule 3: Take the negative inverse of that slope ($n = 1/-0.4 = 2.5$) and use it as the exponent in the S.I. formula:

$$\int a^{2.5}\, dt$$

This step appears to follow from postulating the following equivalence

$$\int a^n\, dt = A^n\, T$$

which is correct only when "a" is a constant. Here "a" is the instantaneous level of acceleration, "A" is the effective level corresponding to the ordinate of the tolerance data graph of rule 1. Note that if "a" is a constant, it moves outside the integral sign and the formula becomes equivalent to the Tolerance Limit equation. The equations are equivalent only for this case, the rarely encountered square wave.

Rule 4: Use the constant 1000 from the equation of the line (from rule 1), $1000 = TA^{2.5}$, as the criterion level that should not be exceeded.

Rule 4 is a non sequitur. The development of the S.I. from the tolerance data has the appearance of validating the specific value of 2.5 as the exponent. But it is only an appearance. Suppose the equally plausible other Tolerance Limits of Table 1 were to be used. The exponent could be 1.9, or perhaps 3.2. They are equally plausible as Tolerance Limits, but what rule can we invoke in order to make a choice for S.I. purposes?

A choice really cannot be made, which is clearly demonstrated if we now carry this a significant and crucial step further. Consider, as background, that in the limited region of interest to us, for pulses of 50-100 ms, the tolerable acceleration does not change a lot. This is especially apparent if the tolerance data are looked at on a conventional coordinate plot, such as Fig. 1, rather than in log-log form. The available data are so inadequate that they really do not much contradict this idea; a constant or asymptotic value of acceleration level could be taken as a practical approximation in this limited interval. It is roughly about 45 g. Now let us go through the same rules for constructing a S.I., applying them to this not unreasonable case.

Rule 1: Fit a line. The line passes through approximately 45 g at 0.05 s and also at 0.1 s. Its equation is $45 = AT^{\circ}$.

Rule 2: Slope = 0.

Rule 3: Take the negative inverse of the slope ($n = 1/0 = -\infty$) and use it as the exponent in the S.I. formula:

$$\int a^{\pm\infty}\, dt$$

Actually, the exponent in this case is indeterminate, resulting from the attempt to strike the following equivalence

$$\int a^n\, dt = AT^{\circ} = A$$

Rule 4: Use the constant from the equation of the line as the S.I. criterion level. The equation $45 = AT^{\circ}$ results in an infinite constant when the attempt is made to obtain the equivalent equation in the form $K = A^n T$.

The above result may seem too far out to be relevant. However, the accepted line, $1000 = TA^{2.5}$, is not that different from a simplifying approximation of zero slope. The severity index formula becomes nonsensical when the rules are applied to a somewhat different but not unreasonable case. Obviously, the "rules" for developing the S.I. are not trustworthy. The arithmetic may be clearer if we assume a slight slope to the tolerance line, say -0.1, corresponding approximately to a 1 g decline per millisecond decade. The exponent $n = 10$, and the constant is 44^{10}. The flatter the tolerance curve, the closer to infinity the exponent gets.

The issue which the Severity Index really seeks to resolve is the indefiniteness of the acceleration level, "A." When the pulse is square-topped, there is no problem: A is the amplitude of the top. But what is the acceleration level A when the acceleration pulse is a triangle? The peak? Suppose the triangular wave were to have the same area, the same delta-V as the square-top wave. Are the two waves equivalent in their effect, or is the triangular wave twice as severe? The S.I. formula above can provide no guidance in the matter, which is ultimately one of empirical determination.

Effective Acceleration versus Severity Scaling

The tolerance limit graphs and the formulas based on them suffer from a significant cart-before-the-horse problem because the scale of the ordinate variable "A" is ambiguous in denotation, violating a fundamental prerequisite that measurement must be in consistent terms. It is difficult to know where to plot a point on a graph of acceleration versus duration if the rules for determining the numerical value of the acceleration are vague or ambiguous. ("Consistency" does not mean an impossible freedom from random or experimental error, but uniformity of meaning.) The requirement of consistency precedes the establishment of a reliable tolerance limit. The graphs shown previously—which are the only significant data on this subject—do not satisfy even their avowed purpose of displaying an isoseverity boundary because real-world deceleration pulses do not conform to the narrow definition of acceleration level by which those graphs were plotted.

The scaling of severity and a consistent measure for "effective acceleration" can be easily confused. Severity scaling refers to the idea that a change in severity index level would connote a corresponding increment in real-world injury hazard. Consistency in measuring acceleration level, on the other hand, means that waveforms of quite different shape should be indicated by the same numerical value if they cause the same amount of injury. A consistent measure does not have to produce numbers proportional to the level of injury, only that equal measures refer to equal degree of injury. In only a very limited sense does the establishment of a consistent measure in itself yield a severity scale. Severity scaling goes a big step further, but it is clear that consistency is a prerequisite. Appendix A develops a paradigm for severity scaling based on the proportion of exposed persons who show significant response (say, fatality) to graduated levels of impact.

Summary - Tolerance data exist in the form of graphs showing the tradeoff between acceleration level and pulse duration for a threshold degree of injury. Equations can be derived from fitted approximations to the threshold contour on these graphs, the most familiar criterion being describable by the formula

"$1000 = TA^{2.5}$." The formula for the tolerance limit does not provide any basis whatsoever for the scaling of severity, that being a separate issue requiring presently unavailable data in which incremental degrees of injury would be correlated to incremental levels of input. An even more fundamental defect exists, however, rendering not only the severity scaling an open issue, but frustrating as well attempts to apply the simpler idea of the tolerance limit. That defect is the absence of a measure for acceleration level which is consistent regardless of waveform, the "effective acceleration." The formula for the tolerance limit does not provide any basis whatsoever for establishing a consistent measure of acceleration level either, that too being a separate issue requiring determination of the range of waveform variation resulting in equal degrees of injury.

The remainder of this paper will deal somewhat more closely with the SAE Severity Index, and with an alternative approach based on brain response.

SAE Severity Index

Some of the difficulties with the SAE Severity Index have been pointed out, for example, by Slattenschek and Tauffkirchen (6), by Brinn and Staffeld of Chrysler (7), and by Fan of Ford in two technical reports (8, 9). Different waveforms having the same duration and the same average acceleration yield quite different values on the S.I., as shown in Table 2. The illustrations in Fig. 5 show that the SAE S.I. deviates quite markedly from the Wayne State Tolerance Curve; the discrepancy can exceed 50%.

Table 2 - Severity Index S for Pairs of Values b_{mo} and t_{so} of the Wayne State Curve in Rectangular, Triangular, and Sinusoidal Impact Curves

Wayne State Curve		Severity Index for Different Acceleration Pulses		
t_{so} S	b_{mo} g	Rectangular $\square$ $S_{\square} = b_{mo}^{2.5} \cdot t_{so}$	Triangular S_{Δ} $S_{\Delta} = 1.62.S_{\square}$	Sinusoidal $S_{\frown}$ $S_{\frown} = 1.40.\ S_{\square}$
0.0025	170	942	1522	1322
0.0050	100	500	808	702
0.0100	71	425	687	596
0.0200	56	469	758	658
0.0250	54	536	866	752
0.0400	52	780	1261	1095
0.0500	51	929	1501	1303
Nominal value		–	1000	–

Source: Ref. 6

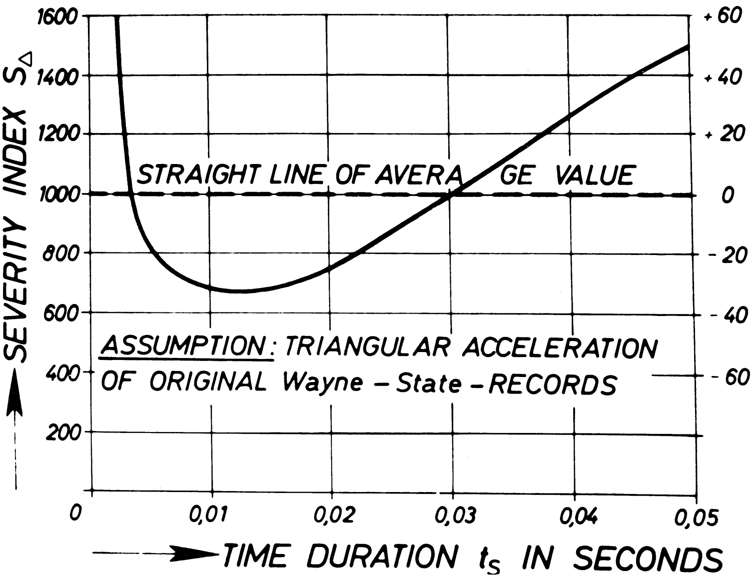

Fig. 4 - Straight line for average value and actual slope of Severity Index S_Δ in the range $2{,}5 \leq t_s \leq 50$ ms. Average value S_Δ = 1,000 equals the theoretical value on the basis of the Wayne State curve. Deviation $A = \frac{S_\Delta - 1{,}000}{1{,}000} \cdot 100\%$

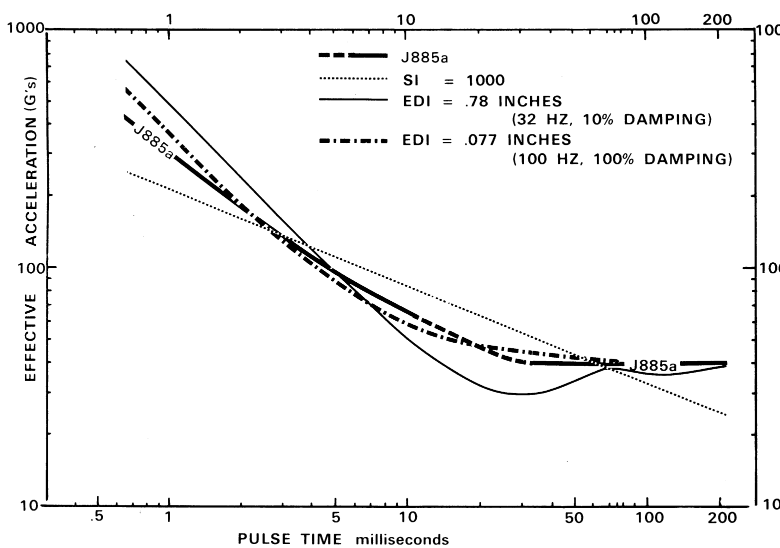

Fig. 2 - Fit of the SI and EDI to the head tolerance curve of SAE Information Report J885a. (Effective acceleration is employed here as defined by Patrick (8). The Severity Index is only recommended for pulse durations ranging 1-50 ms. It has been extended here to cover the range depicted by the curve of J885a)

Fig. 5 - Comparison of S. I.

The SAE S.I. duplicates the Tolerance Limit formula exactly when square waves are involved, so the idea of its general validity tends to be reinforced. But it is only an incidental correspondence, somewhat analogous to the momentarily correct reading displayed at the same time everyday by a stopped clock. For triangular waves, however, the SAE S.I. gives a reading 1.62 times as large as that for the corresponding square wave (that is, square wave of same duration and area). Some would say that is desirable, that peaked waves should read more than flat-topped waves. But, how much more? Earlier in this report it was seen that according to the logic of S.I. definition, any number of equally plausible indexes might be proposed—even one with infinite exponent! Previously, exponents ranging 1.9-3.2 were shown to be consistent with the Wayne State Tolerance Limit, and the extension of that Tolerance Limit in the narrow interval between 50 and 100 ms (the zone of greatest interest to us) has an equation with an exponent of 6. If the Severity Index were to take on the exponent of the associated Tolerance Limit line in each case, the ratio of resultant readouts for triangular versus square waves (of equal area and duration) would be as follows:

n	Ratio
2	1.33
2.5	1.62
3.6	2.64
5	5.35

An example was discussed earlier where a test result had the combination of acceleration and duration which happened to fall at the crossover point of two

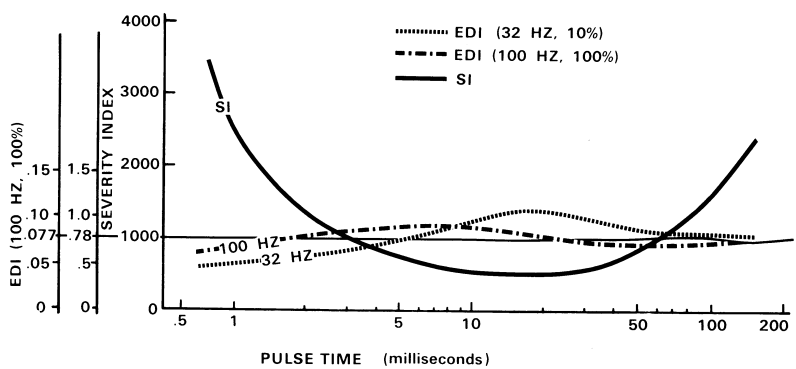

Fig. 4 - Damage indices calculated for each point on the head tolerance curve of J885a and Wayne State Tolerance Curve

alternative and equally plausible Tolerance Limit lines; one line had the 2.5 exponent, the other was a slightly shallower hypothetical line with a 3.6 exponent. That test point would produce S.I. values exactly equal to the criterion value appropriate to each curve (1000 in the first case, some other equally definitive constant in the second) if the wave were rectangular. But if triangular, the S.I. would read 1.62 times as great in the first case, and 2.64 in the second. It is the same pulse and it has not moved on the tolerance limit graph, but if the wave was triangular it would produce a reading which is 63% larger (that is, 2.64/1.62 = 1.63) on the "3.6" S.I. than on the "2.5" S.I.

If the SAE Severity Index cannot be justified on the basis of tolerance limit data, as was amply documented in preceding sections of this paper, then what should replace it? There is no ready answer to that because the human tolerance data required to establish a consistent definition of "effective acceleration" are not available, as was discussed earlier. If a consistent definition were available, the tolerance limit graphs would make more sense as far as practical application goes, and tolerance limit formulas based on them could be used. The tolerance limit formulas developed in this paper were shown to be valid in form, even if there is inadequate data to assure certainty in the precise parameter values, unlike the SAE Severity Index which does not follow from its premises even in form.

What is the SAE Severity Index, really, if it cannot be justified on the basis of the tolerance limit data, and especially since it does happen to be equivalent to the Tolerance Limit formula in the incidental instance of square waves? To clear up the last part of the question first, it is easy to see that for square waves, instantaneous acceleration is constant (at level A), so that

$$\int a^n \, dt = a^n \int dt = A^n \, T$$

The same kind of relation, of course, holds for the other indexes that might be contrived from the parameters listed earlier. For example,

$$9580 = \int a^{3.2} \, dt = a^{3.2} \int dt = A^{3.2} \, T$$

This fortuitous relation no longer holds if the waves are not square waves. The integrand is no longer a constant so it cannot be moved ahead of the integral sign. In puzzling over what the S.I. really is, it seemed plausible to consider it to be a variable that reflects some unique definition for the "effective acceleration." To investigate that, consider the general form that the "effective acceleration" might be postulated to take, for example

$$A = \left(\frac{1}{T} \int a^m \, dt \right)^r$$

where m and r are unknown (and strictly speaking, unknowable with the present absence of biomechanical tolerance data that would pertain to a wide range of pulse shapes.) It might be concluded from Patrick's explanation (3) and the representation by the University of Michigan group (see Fig. 2) that they consider the "effective acceleration" to be the average waveform level, implying that the constants m and r are 1.0 or something pretty close to it. If both the S.I. formula and the Tolerance Limit equation are to equal the same constant when at the critical level

$$\text{S.I.} = T\,A^n$$

$$\int a^n\,dt = T\left(\frac{1}{T}\int a^m\,dt\right)^{rn}$$

The equality will hold true only if m = n and if the product of rn = 1. Since n is independently determined from the Tolerance Limit line, and might be, for example, 2.5, then r must be 1/n. From this development, the "effective acceleration" implied by the Severity Index formula would be

$$A = \left(\frac{1}{T}\int a^n\,dt\right)^{1/n}$$

It can be seen that the larger n is, the larger the reading will be, and that non-rectangular pulses will be sensitively affected by the value of n. But it is difficult to see why the slope of the tolerance limit line, from which n is gotten, should have any bearing on this quantity at all. The definition of "effective acceleration" should in no way depend on the slope of the tolerance limit boundary; the definition must come first because the data points from which the tolerance slope is extracted cannot even be plotted until the "effective acceleration" can be measured.

We recommend that the waveform average be used for "effective acceleration." While it is probably not strictly accurate, we do not know that it is significantly inaccurate. On the other hand, if we hold out for an "effective acceleration" based on the SAE Severity Index we would have to find some justification for the choice of n = 2.5; it certainly cannot be sustained on the basis of tolerance limit data, nor can any other alternative value.

Such a choice would ultimately be judgmental. Our recommendation to use n = 1 is reinforced by the fact that there are no data to support any other formulation; in fact, it is the way tolerance limit data have been presented by the researchers at Wayne State University and at the University of Michigan. Furthermore, a conservative approach is more realistic because the existing human tolerance data are not known to bear any significant correspondence to the kind of response recorded with crash dummies and other laboratory impacters. And probably of foremost significance, the kinds of S.I. readings currently obtained on vehicle components and systems known from field experience to be highly effective greatly exceed the levels that are supposedly critical.

Implementation of the Tolerance Limit formula,

$$\text{Constant} = \mathrm{TA}^{2.5}$$

using the conventional average waveform level for the "effective acceleration" can be effected as follows: The T.L. formula is equivalent to

$$\frac{(\int a\, dt)^{2.5}}{T^{1.5}}$$

One form of readout is to plot the value of that function as increments of T are accumulated over the pulse duration. The terminal value of this ratio, at the end of the pulse, gives the T.L. value for the whole pulse. However, it is possible that in an extremely irregular pulse, some portion of it could actually have a higher value than the pulse as a whole. In that case, the numerator and denominator would expand at different rates, so the ratio would show a peak. The record is evaluated by noting if the ratio at any point exceeds the criterion value.

There are obvious inadequacies in the above approach, aside from those due to the lack of biomechanical information from which a better estimate of the several exponents appearing in the general form of the Tolerance Limit and "effective acceleration" expression might be gotten. The main inadequacy is that this index represents only an intermediate event in the chain of traumatic events. We need to measure the eventual consequences resulting from such inputs. Of more relevance, then, would be some measure that reflected the response of the brain itself upon head impact. With such a measure, all these concerns for waveforms and severity indices would become unimportant, as we look to brain response for an indication of severity.

Alternative Approaches to an Index

An alternative approach stems from our current understanding, admittedly meager, of the nature of closed head injuries. As described in the paper by Fan (10), other researchers had begun to converge toward the idea that closed head trauma is the result of relative displacement between the brain and the skull. In order to elucidate the mechanism, mathematical models of the brain-skull system have been postulated. Fan has advanced a plausible version of such a model. It is described in both of the referenced reports, and is basically similar to models proposed by Slattenchek and by Brinn (6, 7).

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12. System Effectiveness Study.

APPENDIX
THE SCALING OF INJURY SEVERITY

One way to depict the degree of injury is in conventional bioassay terms, the form which biologists customarily use in describing tolerance to stressing agents. The severity is measured by the proportion of exposed individuals who are affected, say by dying or by showing certain signs of injury. Thus, the LD 50, the "lethal dosage for 50% survival," is commonly encountered as a measure of tolerance. A broader view would be achieved by reporting LD 5, LD 10, LD 25, etc. Indeed, the most appropriate scale for depicting severity would be one whose readings are at least proportional to the scale of survival probability. A scale commonly employed for this purpose by toxicologists and other biologists is the logistic function (11).

$$P = \frac{1}{1 + \exp(-(a + b X))}$$

where P is the probability of being affected in the defined manner (say, a fatality) by an applied stress of magnitude X, and a and b are parameters of the function. The logistic function closely resembles the normal distribution. The tolerance curve estimated by Verne Roberts for closed-head injury and shown in Fig. 31 in Ford's Restraint System Effectiveness study (12) fits the logistic function quite well. Fig. A-1 shows the plot on Berkson's (11) logistic paper. An estimated fit, if X-peak g of a triangular pulse, is provided by

$$P = \frac{1}{1 + \exp(6.66 - 0.037 X)}$$

The exponent of e would be (6.66 − 0.074 X) if X = "effective acceleration" = amplitude of the equivalent square wave.

It might be argued that Roberts' survival function, on which this formula is based, is not realistic because it requires the same amount of acceleration to be lethal regardless of pulse duration (but only for pulses exceeding 20 ms). But the critical amount of acceleration does not vary a whole lot, in most reported findings, over the limited range of pulse durations which 30 mph automotive impacts will entail, some 40-100 ms.

However, if a declining tolerance with longer pulses is still deemed more realistic, the logistic function can be modified to take that into account. A plausible adjustment would be to allow the whole logistic function to shift gradually downward in accordance with the acceleration-duration Tolerance Limit discussed in the main text:

$$1000 = T A^{2.5}$$

The critical question which now arises is this: to what LD percentile (or P, in the logistic equation above) does the Tolerance Limit curve correspond? It is likely that it corresponds to a very low LD percentile in view of the data on which that limit was based—the Wayne State experiments, survived free falls, voluntary sled tests, etc. Patrick, et al., say that the WSU Tolerance Limit "is

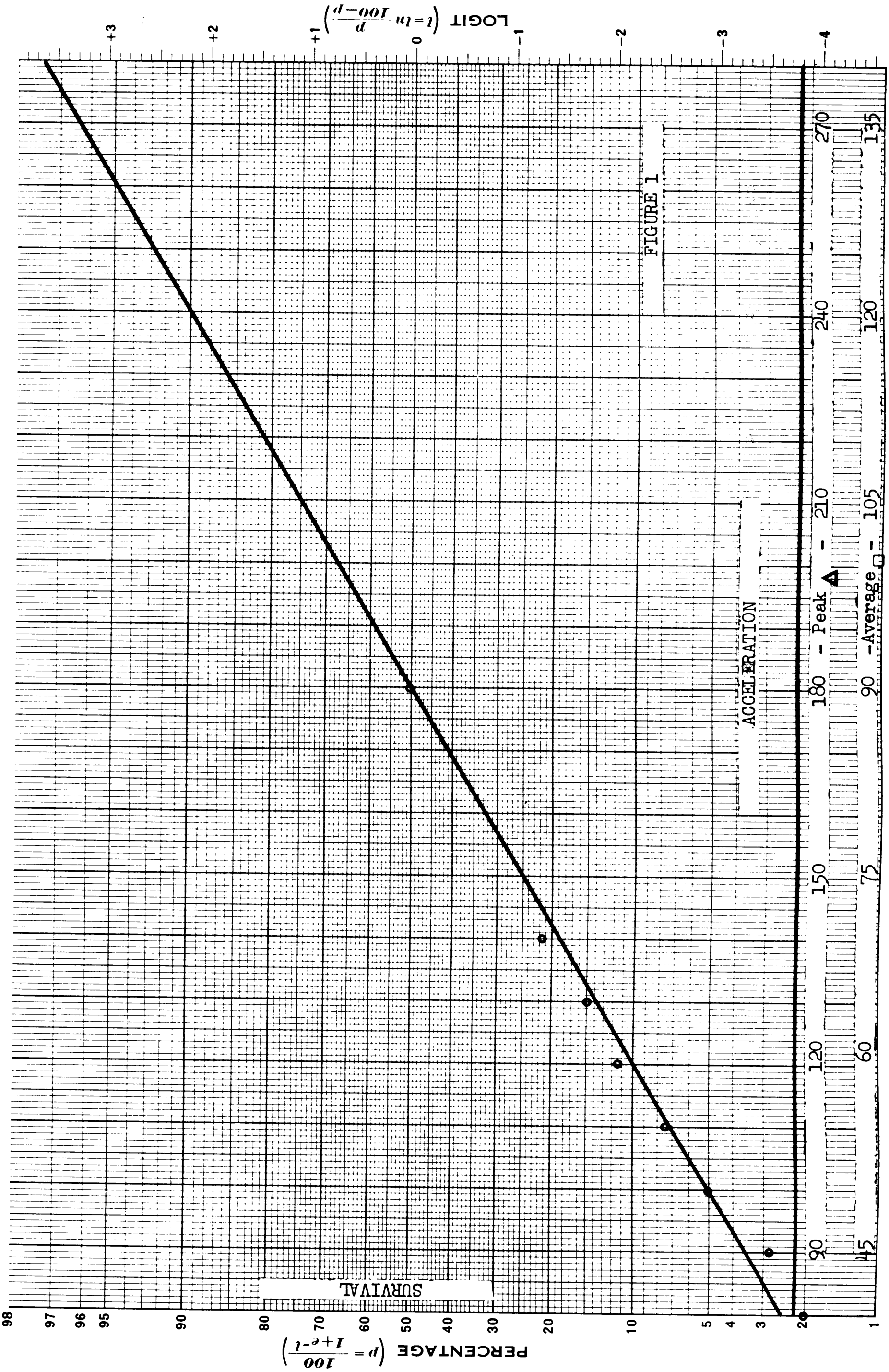
FIGURE 1
ACCELERATION
- Peak
-Average
SURVIVAL
LOGIT
PERCENTAGE

Fig. A-1

based on a reversible concussion with no after effects" (3). For purposes of further exposition here, it is assumed that the Tolerance Limit curve corresponds to LD 5, the chance of fatality is 5%. While hairline fractures may appear and some degree of concussion may result—assuming no concentrated loads that would cause depressed injury—fatality is likely to be infrequent.

The choice of LD 5 would be consistent with both the Tolerance Limit and with Roberts' survivability curve if we posit the following: Roberts' curve really does not apply uniformly regardless of pulse duration but will be assumed to apply only midway between 20 and 100 ms, at which point the 5th percentile of the survivability curve intersects the (1000; 2.5) Tolerance Limit; the whole survivability curve shifts according to the (1000; 2.5) Tolerance Limit curve; "effective" g is one-half the triangular peak value. On that basis, waves of approximately 45 g (effective) and 50 ms would be 5% lethal, 90 g 50% lethal, and 130 g would be 95% lethal. The transition between 5 and 50% fatality spans 45 g. If we assume the same spread of g for all the pulse durations that might occur in the range being considered, the logistic function becomes:

$$P = \frac{1}{1 + \exp\left(3.33 + 0.074\left((1000/T)^{0.4} - \overline{X}\right)\right)}$$

where:

T = duration, s
$\overline{X}$ = "effective acceleration"

Fig. A-2 shows a family of logistic functions corresponding to this expression. An alternative view is seen in Fig. A-3, where an isoseverity family is plotted in the more familiar acceleration-duration graph.

Another question, among others, is this: is the survivability function really symmetric with respect to "effective g" as seen in these figures? In bioassay work, organisms often respond, in logistic curve manner, to the logarithm of the stressing agent. (Because the logarithmic relation is so common, special graph paper is available for plotting the logistic function that way.) In the formula above, X has been taken alternatively as peak g and as average g—should it perhaps be log g, or $g^{2.5}$? No information exists which would allow for a definitive choice. In view of the Wayne State data being expressed as "effective acceleration," and the military human exposure data being very equivocal (square-top time-acceleration *sled input*, not occupant response) it would seem that the most plausible hypothesis at this point is that the survivability function is symmetric and that "effective g," the pulse average measured on the head, is the appropriate pulse descriptor.

Still another question, and perhaps the most crucial of all, bears upon practical application in distinction to scientific findings. The numbers that have been used in this report are supposedly normative, they deal with values that are observed or are at least presumed to be observable in real people—not in the mechanical devices of the vehicle testing workshop. The latter require a *calibration curve* to relate their readings to the normative population figures. (However, do not lose sight of the fact that no true bioassay type experiments have ever been done, or could ever be allowed to be done, with live people. So, a second source of validity erosion occurs in the inference that embalmed cadavers

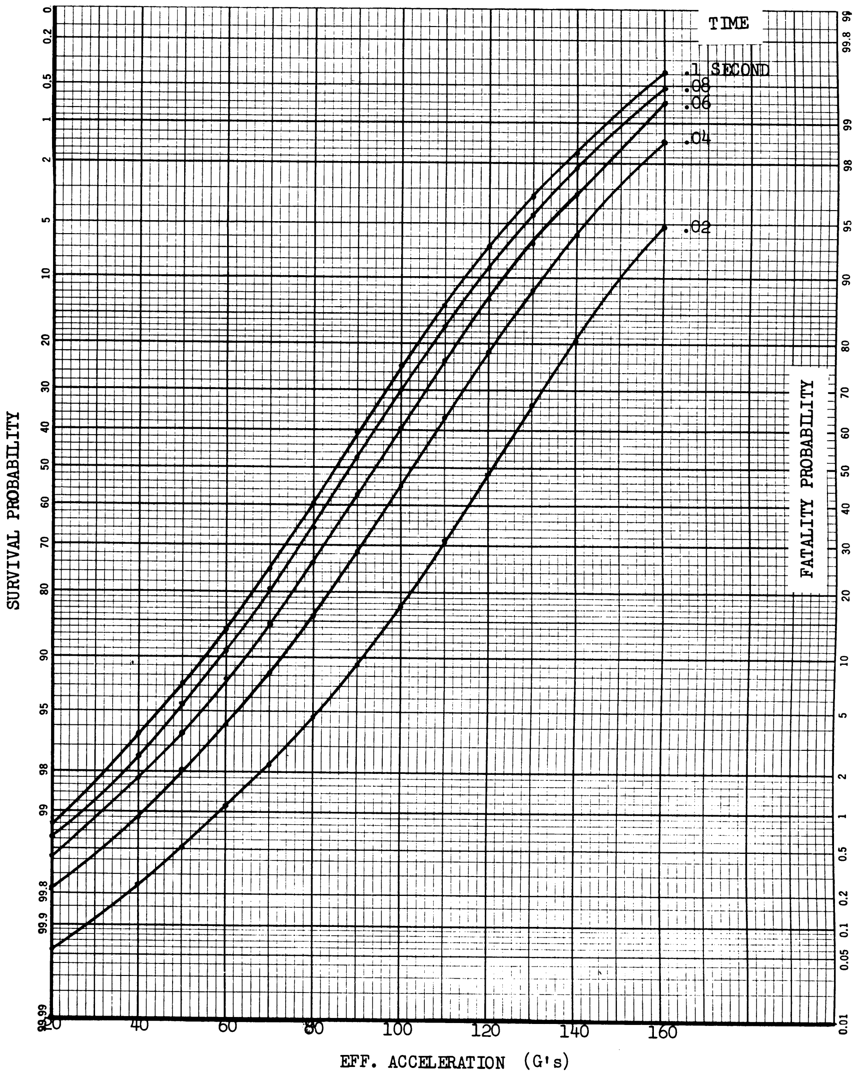

Fig. A-2

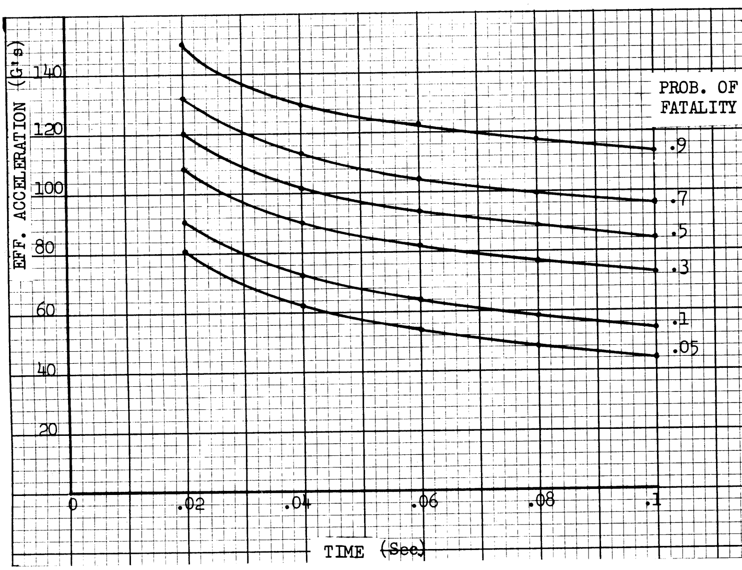

Fig. A-3 - Human tolerance to head acceleration

or live dogs produce responses that are one-to-one normative to people.) Owing to quite different impedances in the testing workshop devices as compared to humans, and to the observed differences in readings recorded when crash test dummies are subjected to the same tests as cadavers, it is likely that the criteria expressed in the functions shown in this paper are quite conservative. Thus, the parameters in the logistic function will no doubt be quite different if that function is supposed to relate the LD of real people to the readings gotten in crash dummies exposed to the same crash. And that relationship is the only valid one.